Chapter-10

Gravitation

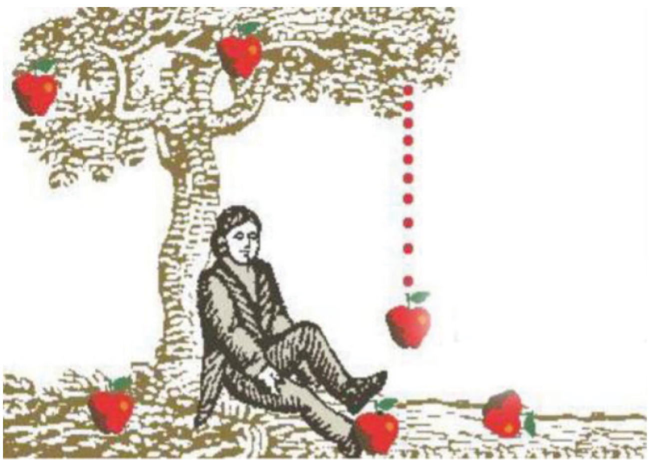
Activity 10.1

- On leaving a stone from hand it automatically falls down on the earth surface.
- Some force is to be applied if we want to throw up the same stone in the upward direction.

In the last chapter we have studied that force is required to change the movement or direction of motion of an object or body. On considering the force required in the above activities we understand that on leaving any object freely it moves towards the surface of the earth. This means, that the earth exerts some invisible force on the object, which is known as the Gravitational Force. When the object is thrown up an external force is applied by the person throwing it up. At that time also earth's gravitational force is working on it and hence its velocity gradually decreases down to zero. Then after it again starts falling down towards the earth surface. This also is an example of earth's gravitational force. Another example of this force is the movement of water drops from the clouds to the earth surface.

10.1 Gravitation :

It is said that once Newton was sitting beneath an apple tree and an apple fell on him. Newton was inspired to find the reason underlying this episode. He thought as to why the apple fell towards earth? Why was it not attracted towards the moon. Thus on the daily life observations Newton proposed the theory of Gravitation.



In this chapter we will study about the Gravitation and the Universal Laws of Gravitation. We will also consider the movement of object under the influence of the gravitational force, the change in weight of objects and why do objects fly in the space.

Let us see as to how circular motion takes place.

Activity 10.2 :

- Tie a small stone to a piece of a thread.
- Move the stone on a circular path by catching hold of the other end of the thread, as shown in fig.
- Observe the direction of the movement of the stone.
- Now leave the thread and observe the direction of movement of the stone.

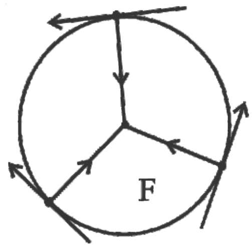


Fig. 10.1

Before we left the thread the stone was moving in a circular path. At that time it experienced a force towards its center which is known as the Centripetal Force. This force is responsible for the circular motion of the objects.

On leaving the thread the stone proceeds in a simple line and then moves in a free manner i.e. the circular motion ends on removing the Centripetal Force.

The movement of moon around the earth is due to the presence of centripetal force. This centripetal force is derived from the earth's gravitational force.

Gravitational Force occurs naturally between all the objects of the world, but we are unable to experience the gravitational force working between the normal objects. Since the mass of celestial bodies is more this force becomes effective and controls their motion.

It was in fifth century that the Bhartiya astronomer Aryabhatt propounded the geocentric model to understand the movement of planets. Nearly 500 years before Newton and Kepler, Bhaskaracharya in the grah-ganit section of his famous work Siddhantsiromani had discussed the gravitational power of earth and the planetary movements, in detail. Bhaskaracharya calculated the radius (R) and circumference ($2\pi r$) of the earth. The western scientist Copernicus (1473-1543) propounded the model of the movement of celestial bodies on the basis of Aryabhatt's vision. Even Kepler and Galileo also worked out some laws to clarify the understanding about the movement of the planets.

The famous seventeenth century scientist Isaac Newton (1642-1727) gave the Laws of Motion and the Universal Law of Gravitation. These were based on substantial scientific logic and were proved mathematically. The contributions of these scientists in the sphere of physics were very important.

10.2 Universal Law of Gravitation :

Each particle (body) of the universe exerts a force of attraction towards itself on every other particle (body) which is known as the Gravitational Force.

According to the Law of Gravitation "the force of attraction between two particles of matter or bodies is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them. The direction of this force is the same as the direction of the line joining the two particles".

Suppose two bodies of mass M and m are placed at a distance d from each other. Then the gravitational force F working between will be

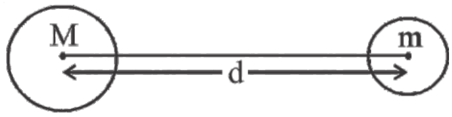


Fig. 10. 2

(1) $F \propto Mm$ .... (i)

(2) $F \propto \frac{1}{d^2}$ .... (ii)

From equation (i) and (ii) we get

$$F \propto \frac{mM}{d^2}$$

or $$F = \frac{GMm}{d^2} \qquad .... (10.1)$$

Here G is the proportional constant known as the Universal Gravitational constant.

The value of G has been worked out to be 6.67×10^{-11} Newton–meter^2–Kilogram^2, from various experiments. Its value does not depend on the nature of the particles, medium, time, temperature etc., i.e. it remains the same at every place. Hence it is known as the Universal Constant. The earth also attracts things towards itself. This, earth's force of attraction is known as the Force of Gravitation.

Derivation of the unit of G :

From equation 10.1 : $Fd^2 = GMm$

or $$G = \frac{Fd^2}{Mm} \qquad .... (10.2)$$

When we place the units of F, d and Mm in equation (10.2), the SI unit of G obtained will be

$$\frac{Nm^2}{Kg^2}$$

Many phenomenon can be easily interpreted by the Law of Gravitation. Some of the important ones includes :

(1) The force that binds us to the earth
(2) Movement of planets around the sun
(3) Movement of the moon around the earth
(4) Occurrence of tides in the sea.

Example 10.1 : The mass of the earth is approximately 6×10^{24} Kg and that of the moon is approx. 7.4×10^{22} Kg. If the distance between the earth and the moon is 3.84×10^5 Km, then calculate the force exerted by the earth on the moon

Here $G = 6.7\times10^{-11} \frac{Nm^2}{Kg^2}$

Solution :

Mass of the earth $M = 6\times10^{24}$ Kg

Mass of the moon $m = 7.4\times10^{22}$ Kg

Distance between the two $d = 3.84\times10^5$ Km

$$G = 6.7\times10^{-11} \frac{Nm^2}{Kg^2}$$

$$d = 3.84\times10^5 \text{ Km} \quad = 3.84\times10^5\times1000 \text{ m}$$
$$= 3.84\times10^8 \text{ m}$$

The force applied by the earth on the moon

$$F = \frac{GMm}{d^2} = \frac{6.7\times10^{-11}\times6\times10^{24}\times7.4\times10^{22}}{3.84\times10^8\times3.84\times10^8}$$

$= 20.17\times10^{19}$ N

$= 2.02\times10^{20}$ N

Example 10.2 : A ball of 40 Kg mass experiences a gravitational force of 0.25×10^{-6} Kg weight by another ball of 80 Kg mass. The distance between the centers of the two balls is 30 cm. If the gravitational acceleration $g = 9.8 m/s^2$ then calculate the gravitational constant.

Solution :

Here $m_1 = 40$ Kg

$m_2 = 80$ Kg

$W = F = mg = 0.25\times10^{-6}\times9.8$ Newton

$r = 30$ cm $= 0.3$ meter

$$F = \frac{G\, m_1 m_2}{r^2}$$

$$G = \frac{Fr^2}{m_1 m_2}$$

$$= \frac{0.25\times10^{-6}\times9.8\times0.3\times0.3}{40\times80}$$

$$= \frac{225\times98\times10^{-11}}{32\times100}$$

$= 6.89x10^{-11}\ Nm^2/Kg^2$

Activity 10.3

(i) When a stone is thrown up it moves up to a certain height and then starts falling down.

(ii) A stone when left from some height automatically falls downwards.

You have seen that in the first case the stone moves up for some distance. This distance depends upon the force applied by you. In this situation the direction of the movement of the stone is opposite to the direction of the earth's gravitational force. Therefore, after attaining a certain height the stone starts falling down.

In the other situation the stone automatically falls down from some height when left. This is known as the free fall. In this case the direction of the movement of the stone and that of the force of gravitation, both are the same.

In both the above cases the velocity of the body changes thus its movement is under the influence of the gravitational acceleration.

10.3 Gravitational Acceleration :

When the change in an object's velocity i.e. acceleration, is due to the earth's force of gravitation, it is known as the Gravitational Acceleration. It is denoted by g and its unit is ms^{-2}.

From the second law of motion

Force = mass × acceleration

or $F = mg$ .... (10.3)

On equating equation (10.1) and (10.3) we get

$$mg = G\frac{GMm}{d^2}$$

$$g = \frac{GM}{d^2} \quad \text{.........(10.4)}$$

Here M is the mass of earth and d is the distance of the object from the center of the earth. The value of g decreases with the increase in value of d.

If the radius of earth is R then the gravitational acceleration on the earth's surface g^s will be :

$$g_s = \frac{GM}{R^2} \quad \text{........(10.5)}$$

It is clear from fig. 10.3 that the radius of the earth is more at the equator as compared to that at the poles. Therefore, the value of g will be more at the poles as compared to that at the equator.

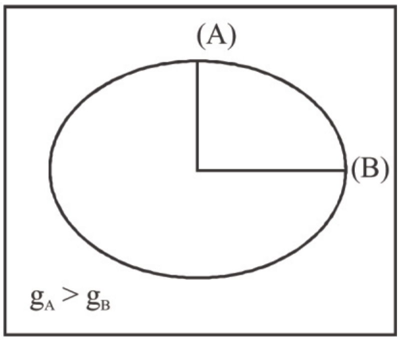


Fig. 10.3

As we move to height h the value of g decreases.

Calculation of the value of gravitational acceleration at the earth surface :

Mass of earth M $= 6\times10^{24}$ Kg

Radius of earth R $= 6400$ Km

$= 6400\times1000$ m

$= 6.4\times10^{6}$ m

From equation 10.5

$$g_s = \frac{GM}{R^2}$$

$$= \frac{6.7\times10^{-11}\times6\times10^{24}}{6.4\times10^{6}\times6.4\times10^{6}}$$

$$= 9.8 m/s^2 \text{ or } ms^{-2}$$

Therefore the value of earth's gravitational acceleration $= 9.8\ ms^{-2}$.

10.3.1 Gravitational acceleration of different planets :

The gravitational acceleration of any planet depends upon its mass and radius. In other words, heavier a planet comparatively more will be its gravitational acceleration.

Activity 10.4

- In the table given below, the mass, radius and gravitational acceleration of various planets have been tabulated.
- Keeping in mind the dependencies of g study the given table and try to answer the questions given below :

Table 10.1

Name of Planets	Mass $X10^{24}$ (Kg)	Radius (Km)	gravitational acceleration (m/s^2)
Mercury	0.33	4879	3.7
Venus	4.87	12104	8.9
Earth	5.97	12756	9.8
Mars	0.642	6792	3.7
Jupiter	1898	142984	23.1
Saturn	568	120536	9.0
Uranus	86.8	51118	8.7
Neptune	102	495.28	11
Pluto	0.014	2370	0.7

[Pluto has been removed from the category of planets in 2006].

Question 1. Which Planet has the maximum value of 'g' and why?

Question 2. What is the value of 'g' on mercury?

Make questions on your own based of this table and discuss with your friends.

> The gravitational acceleration on moon is 1.61 m/s^2 which is about 1/6 of the earth's gravitational acceleration .

10.4 Movement of objects under the influence of the earth's force of gravitation :

It is clear from the equations 10.4 and 10.5 that the Gravitational Acceleration experienced by various objects does not depend upon the mass and shape of the objects. This means that different objects should fall down with the same rate when dropped from the same height. But is it so? Again we will perform an activity to get an answer.

Activity 10.5

- Take some stones, coins, feathers and papers. Throw them down from the terrace of your house and observe. Do all of them reach the earth simultaneously?

> The initial velocity of the free falling objects is zero.

The value of gravitational acceleration 'g' remains constant for a height of some kilometers from the earth's surface. Therefore to study the motion of objects near the earth's surface, the equation for uniform acceleration motion described in the last chapter is made use of.

Here the acceleration a is replaced by the gravitational acceleration g. If the resistance due to air is considered to be negligible, then :

(i) The equations of motion on throwing an object in the upward direction

$$\left.\begin{aligned} v &= u - gt \\ h &= ut - \frac{1}{2}gt^2 \\ v^2 &= u^2 - 2gh \end{aligned}\right\} \quad \text{........ (10.6)}$$

here h is the height of the object from the surface at the given moment of time.

(ii) The equations of motion for a free falling object :

$$\left.\begin{aligned} v &= gt \\ s &= \frac{1}{2}gt^2 \\ v^2 &= 2\,gs \end{aligned}\right\} \quad \text{........ (10.7)}$$

Example 10.3 : A bread from the beak of a crow sitting on the neem tree reaches the earth's surface in 2 seconds. Taking the value of g as $10 m/s^2$, Calculate the following :

(i) What will be the velocity of bread on reaching the earth's surface?

(ii) What will be the average velocity of the bread during these two seconds.

(iii) How high is the crow's beak from the earth?

Solution :

(i) The final velocity [as from equation 10.7]

$$\vec{v} = gt$$

$$\therefore \vec{v} = 10 \times 2 = 20 m/s$$

(ii) Average velocity :

$$\vec{v} = \frac{u+v}{2} = \frac{0+20}{2} = 10 \text{ m/s}$$

(iii) The height of the beak will be the distance travelled, hence

$$s = ½\, gt^2$$

$$= ½ \times 10 \times 4$$

$$= 20 \text{ m}$$

Example 10.4 : An object is thrown up vertically if it reaches to a height of 10m then calculate :

(1) The velocity with which the object was thrown up.

(2) Time taken by the object to reach the highest point.

Solution : Here the maximum height reached

h = 10 m

final velocity v = 0

gravitational acceleration $g = 9.8 m/s^2$.

(1) $v^2 = u^2 + 2gh$

$O = u^2 - 2gh$

$u^2 = 2gh$

$u^2 = 2 \times 9.8 \times 10 = 196$

$u = \sqrt{196}$

$= 14 \text{ m s}^{-1}$.

(2) $v = u - gt$

$O = u - gt$

$u = gt$

$$t = \frac{u}{g} = \frac{10}{9.8} = 1.43 \text{ s}$$

10.5 Mass :

In the last chapter you have studied that Mass is the measure of the inertia of an object. In other words, greater the mass of a body more will be its inertia. The mass of a body remains the same every where; whether it is on earth or in space.

10.6 Weight :

On earth the weight of an object is a type of a force that takes it towards the earth. As per definition the weight of a substance is the force by which it is attracted towards the earth.

Mathematically $F = mg$

$\therefore$ the weight of the body $w = mg$.

Think what is the unit of weight and why

Since the value of g at a given point is constant, therefore, the weight of an object is proportional to it's mass. The mass remains the same with the change in the value of g but the weight changes .

10.7 Weight of an object on the Moon :

Weight of a body on Moon is the force with which the moon attracts it towards itself. The mass of moon is less as compared to that of the earth therefore even the force of attraction will be less. We can say that the weight of an object on moon is relatively less than that on earth.

Since we have seen that the gravitational acceleration on moon is 1.61 m/s^2 which is 1/6 of that present on the earth's surface, whose gravitational acceleration is 9.8 m/s^2, therefore the weight of the object on moon will be 1/6 of its weight on earth.

Weightlessness :

If you have sat in the swings put up in fairs, you must have experienced that when the swing comes down from up we experience reduction in weight.

Basically, the notion of weight is because of the reaction force being applied on the object. If the cable of a lift coming in downward direction beaks, there will be free fall of the lift and we will experience weightlessness, i.e. zero weight.

In case of weightlessness the reaction force. (R) = 0 Thus, artificial satellite continuously falls freely toward the center of the earth and all the things present in it including the organisms, are in a state of weightlessness. In such a condition some very interesting results have been observed. For example :

(1) The water filled in the glass will not fall even if the glass is inverted. This is because on tilting the glass, the water of the glass being heavy will float in the form of drops. Therefore an astronaut cannot drink the water of the glass.

(2) Even the food is to be ingested in the form of a paste directly from the tube by squeezing it.

(3) All the things inside the space craft are afloat.

(4) If the astronaut measures himself with a spring balance, in the space craft, his weight will be zero.

(5) If an object is tied and hung with a string on the space craft, the tension in the string will be zero.

Here the point to be noted is that in an artificial satellite everything experiences weightlessness while the case is not so on a natural satellite. This is because the natural satellite exerts gravitational force because of its mass, which is more. The mass of an artificial satellite however, is less and hence it applies negligible gravitational force and the object is in a state of weightlessness in its environment.

Important Points

1. Some or the other type of force is responsible for the motion of an object. The centripetal force is responsible for the circular motion.
2. Every object of this universe applies a force of attraction, naturally, on every other object, which is known as the force gravitation. All the stars, planets galaxies and small atomic particles are included in the term 'object'.
3. The gravitational force acting between two objects is mathematically represented as under. It is also referred to as the Universal Law of Gravitation

$$F = \frac{GMm}{d^2}$$

where M and m are the masses of the objects concerned and d is the distance between them. G is the constant of proportion whose numeric value is $6.67 \times 10^{-11} \frac{Nm^2}{Kg^2}$

4. If there is a change in the velocity of an object under the influence of the gravitational force, then it is known as the gravitational acceleration.
5. The equation for the value of gravitational

acceleration on the earth's surface is

$$g = \frac{GM}{R^2}$$ where M and R are mass and radii of the earth respectively. The numeric value of the gravitational acceleration on earth surface is 9.8 m/s^2.

6. The value of gravitational acceleration decreases as we move up from the surface or move down to some depth from the surface.
7. The value of gravitational acceleration decreases as we move from the poles to the equator.
8. The value of gravitational acceleration on the surface of any planet depends upon its mass and radius.
9. The largest planet of our solar system is Jupiter and hence its gravitational acceleration also is the maximum.
10. The initial velocity of the freely falling body is zero whereas the final velocity of an object thrown in the upward direction is zero.
11. Every particle, body or object has a mass which is not dependent on the gravitational force while the weight of the object depends on the gravitational force.
12. The relation between the weight and mass of an object is W = F = mg. Here F, the weight of the object is expressed in unit of Newton, m is the mass of the object and g is the gravitational acceleration whose unit is m/s^2.
13. The weight of an object on a planet or satellite depends on the value of its gravitational acceleration. For example the gravitational acceleration on moon is $1/6^{th}$ of that on earth and hence the weight of an object on moon will also be $1/6^{th}$ of its weight on earth.
14. A free falling body is in a state of weightlessness because the reaction force on it is zero.
15. All the things in an artificial satellite are afloat because they are in a state of weightlessness.
16. The understanding about the astronomical objects and their movements has become easy because of the calculation of their gravitational force.
17. The important names of Bhartiya scientist who made contribution in the field of astronomical bodies, gravitational force and planetary movements includes Aryabhatt, Varahmihir and Bhaskaracharya. Western scientists who have made valuable contributions in the field includes : Copernicus, Galileo, Kepler and Newton.

Important Questions

Objective type :

1. The Newton's Law of Gravitation is universal because :
(a) It is always of attraction
(b) It is applicable on all the members and particles of the solar-system.
(c) It is applicable on all the masses, at all the distances and is not influenced by the medium.
(d) None of the above.
2. Which is the force responsible for the circular motion of object :
(a) Gravitational force
(b) Frictional force
(c) Centripetal force
(d) None of the above
3. The value of the Universal Constant G depends upon :
(a) Nature of the particles
(b) Medium present between the particles
(c) Time
(d) Does not depend on anything
4. The weight of a person on earth surface is 60 N then his weight on moon will be :
(a) 60 N (b) 30 N
(c) 20 N (d) 10 N
5. An object of mass m is taken into a very deep mine then :
(a) Its mass increases
(b) Its mass decreases
(c) Its weight decreases
(d) Its weight increases
6. On doubling the distance between two masses, the gravitational force between them :
(a) Remains unchanged
(b) will become one fourth
(c) will be reduced to half
(d) will double itself

Very short answer type questions :

1. From where do the satellite derives the

centripetal force required by it to revolve round the planets?

2. Can the gravitational mass of a body be measured in an artificial satellite?
3. What will be the change in the gravitational force between two masses if the distance between them is doubled?
4. If the mass of a body is 10 Kg then what will be its weight on the earth surface?
5. Write the formula of the gravitational force between two bodies.
6. On which principle does a ball pen work?
7. A person can jump on the Moon for a greater height. Why?
8. Two bodies of 1 Kg each are at a distance of 1 meter from each other. Write the value of gravitational force between them.
9. Why does the moon not fall down on experiencing the earth's gravitational force?
10. What is the main reason for oceanic tides?

Short answer type questions :

1. Write the Universal Law of Gravitation.
2. On the earth's surface the value of g is maximum at what point and why?
3. Write the value of the universal constant of gravitation along with its unit.
4. What is Gravitational Acceleration? Write its formula.
5. Clarify the difference between the mass and weight of an object.
6. What is meant by Free Fall? Give examples of free fall.
7. What will be the change in weight of a person moving from the poles of the earth to its equator and why?
8. What is weightlessness? Give two interesting examples of weightlessness.
9. Write the three equations of motion for a freely falling body. Also write the meaning of the symbols.
10. An object is thrown in the upward direction with a velocity u so that it attains a height of h. Write the three equations for the movement of the object.
11. What are the difficulties faced by an astronaut due to weightlessness.
12. Who were the ancient Bhartiya astronomy scientist prior to Kepler and Newton? Write their contributions in the field of gravitation.
13. The objects in an artificial satellite are in a condition of weightlessness while it is not so on a natural satellite. Why?

Essay type questions :

1. Find the value of gravitational force between two balls each having a weight of 10 Kg placed at a distance of 50 cm from each other.
2. Calculate the gravitational force on a 40 Kg object placed on the earth's surface. The radius of the earth R is 6400 Km and its mass is 6×10^{24} Kg.
3. A stone is thrown up in the air with an initial velocity of 20m/s. Taking $g = 10m/s^2$ calculate the following :
   (i) time taken by the stone to attain the maximum height
   (ii) distance travelled by the stone.
4. Find the Gravitational Acceleration on the moon surface if the mass of the moon is 0.073×10^{24} Kg and its radius is 1738 Km.
5. A stone is thrown from a 125 m high tower.
   Calculate (i) time taken to reach down
   (ii) final velocity of the stone
6. If the radius of the earth becomes half of the present radius then the mass will become 1/8th. What will be the g of this earth of half the shape?

Answer Key

1. (c) 2. (a) 3. (d) 4. (d) 5. (c) 6. (b)